Binary Black Hole Simulations

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Numerical Relativity

Astonishing progress in the last year

- Pretorius has stable black hole orbits
- UTB and NASA bring stable codes to the community
  - ▷ "Moving Punctures"
  - ▷ Small modifications of evolution [UTB] or gauge equations [NASA]
  - ▷ Simple implementation
- Code crashes are (pretty much) history
  - ▷ I still have occasional crashes, but they are rare
  - ▷ Frequently there is an easy workaround,
    i.e. a little more dissipation, moving the outer boundary further out, . . .

Unequal-Mass Simulations

Study different mass ratios $q = M_1/M_2$ (comparable masses)

- Look for effects in waveforms
- Study recoil velocities from full numerical simulations

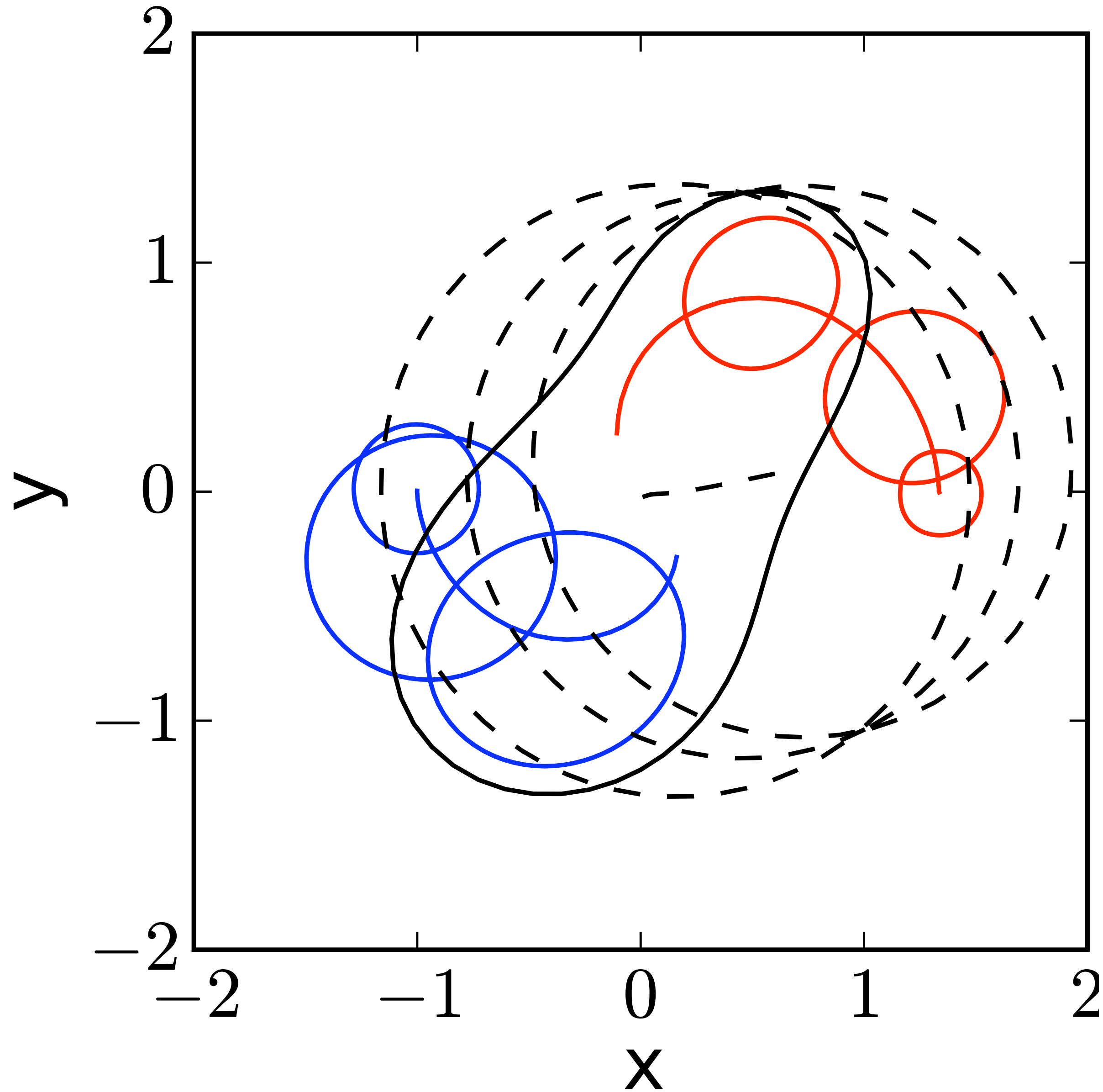
Initial data: Increase one of the bare mass parameters for QC-0

- Numerical convenience rather than astrophysical realism
- Of course more stuff changes than just the mass ratio ...

Time to common apparent horizon

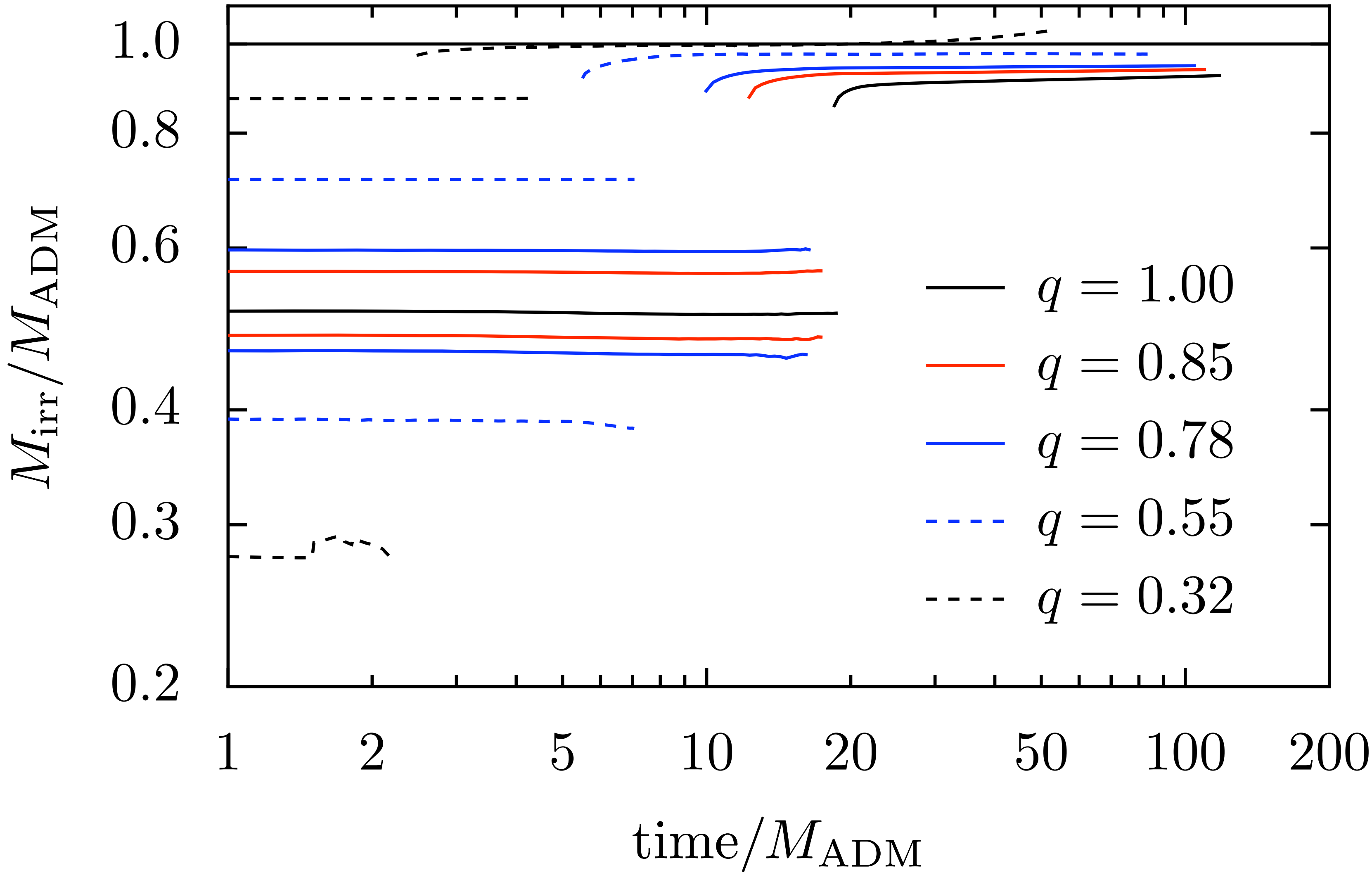
$q \equiv M_1/M_2$	t_{AH}/M_{ADM}
1.00	18.4
0.85	12.2
0.78	9.9
0.55	5.5
0.32	1.5

Apparent Horizon Snapshots [q=0.78]



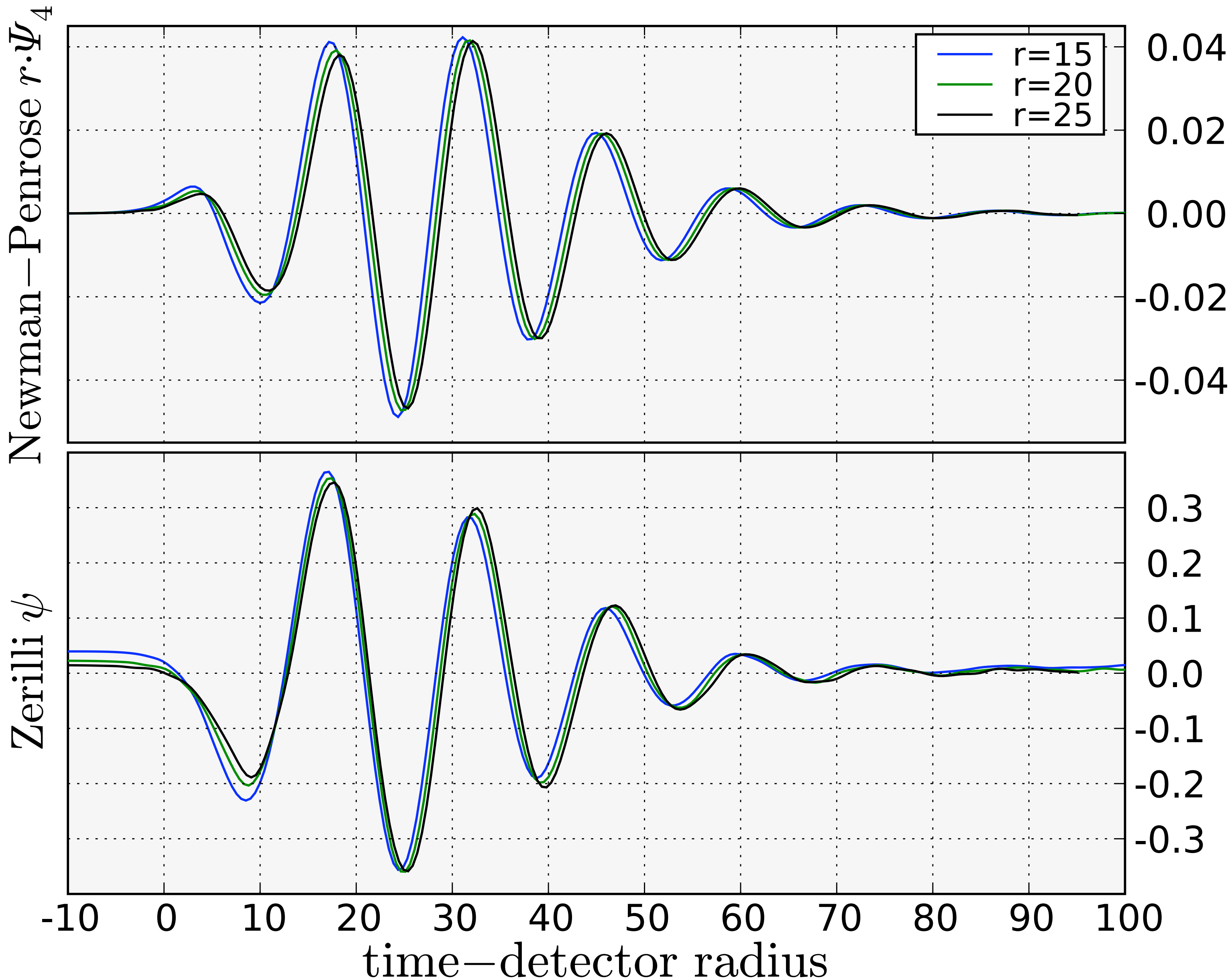
$t = \{0, 4.6, 9.9, 40, 80, 105\}\, M_{\mathrm{ADM}}$

Irreducible Mass of the AH



q : mass ratio

Waves: Zerilli ψ & Newman-Penrose Ψ_4 [q=0.85]



Ψ_4 provided by Ian Hinder and Tanja Bode

Getting the Numbers out of Zerilli

Extract radiation using Zerilli $\psi_{\ell m} \to h_{ij} \to$ estimate radiated E, J, V

$$\frac{dE}{dt} = \frac{1}{16\pi} \sum_{\ell m} \frac{(\ell+2)!}{(\ell-2)!} \left[\left| \frac{d\psi^{+}_{\ell m}}{dt} \right|^2 + \left| \psi^{\times}_{\ell m} \right|^2 \right]$$

$$\frac{dJ}{dt} = \frac{1}{16\pi} \sum_{\ell m} \imath m \frac{(\ell+2)!}{(\ell-2)!} \left[\frac{d\psi^{+}_{\ell m}}{dt} \left(\psi^{+}_{\ell m}\right)^* + \psi^{\times}_{\ell m} \int_{-\infty}^{t} \left(\psi^{\times}_{\ell m}\right)^* dt' \right]$$

$$h_+ - \imath h_\times = \frac{1}{r} \sum_{\ell,m} \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \left[\psi^{+}_{\ell m} - \imath \int_{-\infty}^{t} \psi^{\times}_{\ell m} dt' \right] {}_{-2}Y^{\ell m}(\theta, \phi) + \mathcal{O}\left[\frac{1}{r^2}\right]$$

$$\frac{dP^k}{dt} = \frac{r^2}{16\pi} \int_S \left[\left(\frac{dh_+}{dt} \right)^2 + \left(\frac{dh_\times}{dt} \right)^2 \right] n^k d\Omega$$

Results from radiation extraction

q	$\Delta E/M_{ADM}$ [%]	$\Delta J/J_{ID}$ [%]	V (km/s)
1.00	2.7 ± 0.4	15 ± 3	1 ± 1
0.85	1.7 ± 0.1	10 ± 0.4	49 ± 11
0.78	1.1 ± 0.4	7.4 ± 0.4	69 ± 19
0.55	0.4 ± 0.1	2.6 ± 0.3	82 ± 27
0.32	0.05	0.4	25

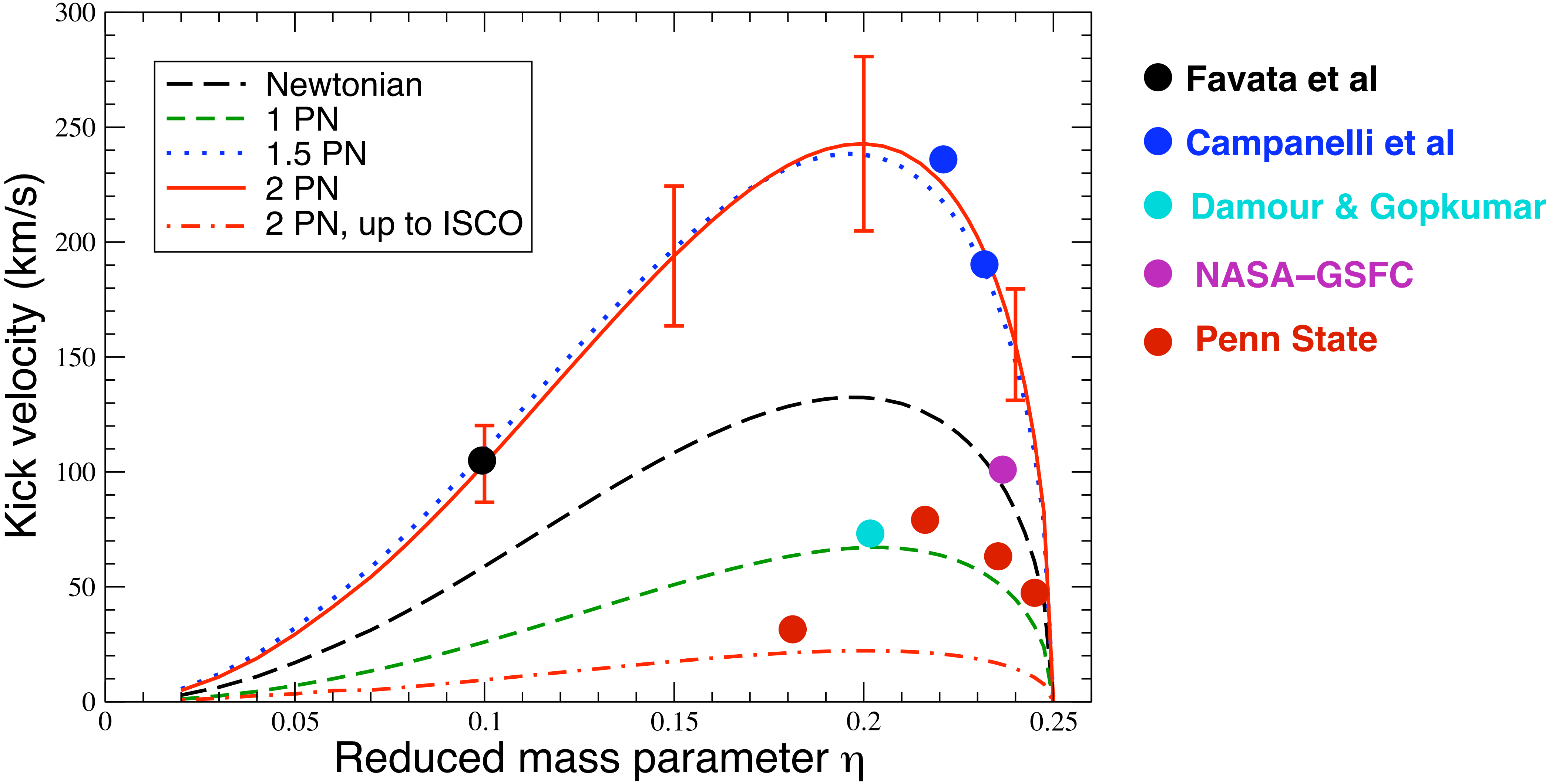
Error in radiated energy and radiated angular momentum not phase dependent

- ΔE, ΔJ depend only on independent $\psi_{\ell m}$ modes
- Peaks dominate and in particular ψ_{22} dominates

Kick velocity is much more tricky

- Overlap between modes is crucial
- Fully exposed to relative phase error between $\psi_{\ell m}$ modes
- Waveforms were truncated to $T = [7, 70]M$ for recoil velocity

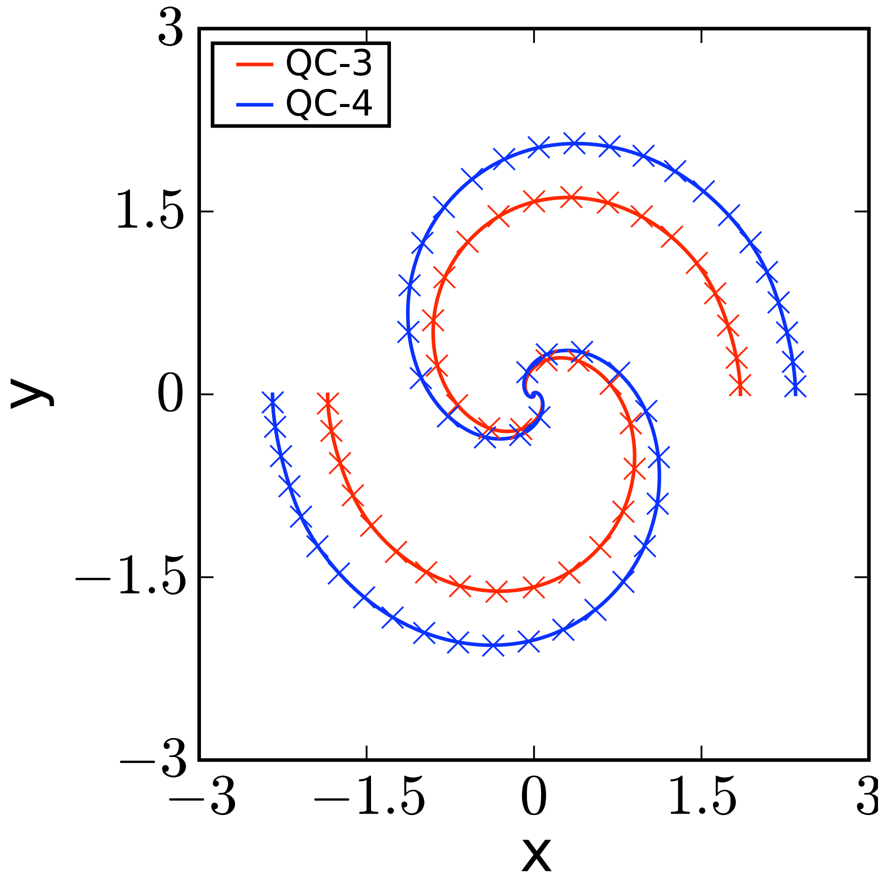
Different recoil estimates

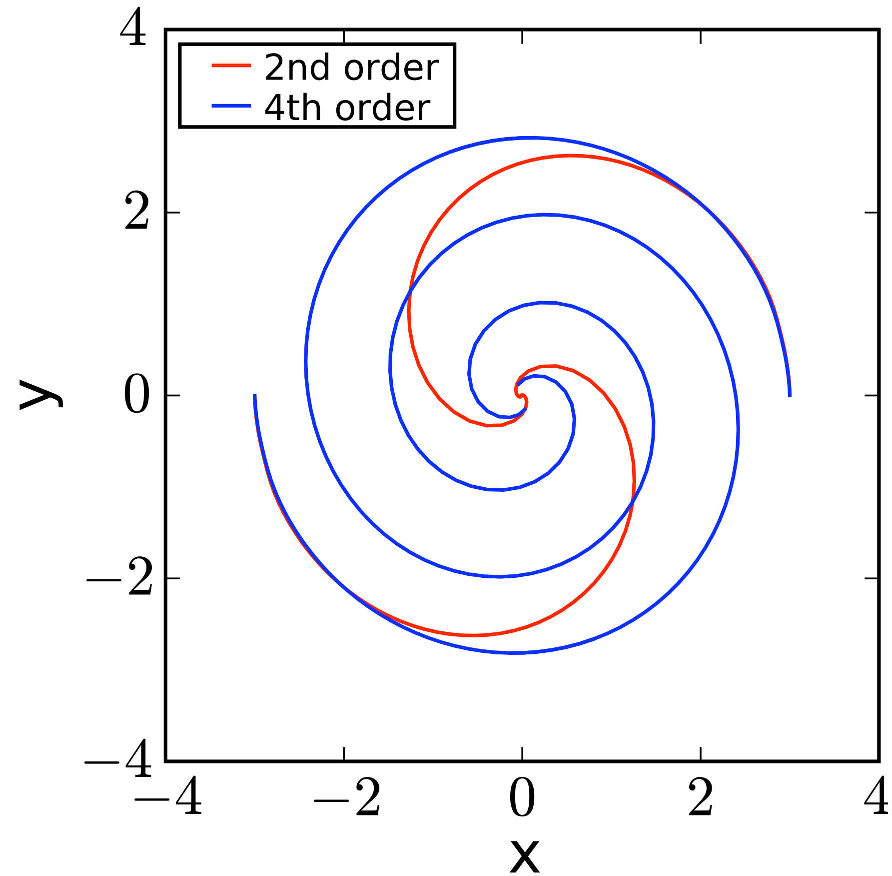


$$\eta = m_1 m_2/(m_1 + m_2)^2$$

Blanchet, Qusailah, Will 2005

Getting to Further Separations





- Comparison of AH center and puncture location via $\partial_t x_P^i = -\beta^i(x_P)$
- different finite difference orders for advection terms ($\beta^i \partial_i$) only

Outlook & Conclusions

Current BH evolution recipe:

$\boxed{\text{Move the holes, don't bother with excision}}$

Many groups now have working codes

- Independent checks of results
- Comparison of different codes

Lots of stuff will be studied in the next 1-2 years

- Thorough study of unequal-mass systems and spin
- Recoil velocity in particular is sensitive quantity
- More numerical experience is needed

I still see crashes, but much less fine-tuning needed

END

This is the End.

Appendix

Appendix starts here.

PSU Implementation

Basically follow NASA prescription [gr-qc/0511103]

- Easier to implement than UTB
- Gauge modification for standard Γ-Driver
- New advection term $\beta^i\partial_i\tilde{\Gamma}^i$ (removes "puncture memory effect")

$$\partial_t\beta^i = \frac{3}{4}\alpha B^i, \text{ with } \partial_t B^i = \partial_t\tilde{\Gamma}^i - \beta^j\partial_j\tilde{\Gamma}^i - \eta B^i$$

- Use "1+log" α-evolution, i.e. $\partial_t\alpha = -2\alpha K$
  - ▷ (i.e. no $\beta^i\partial_i\alpha$ term) unlike NASA and UTB

Initial Gauge

- Initial shift ($\beta^i = 0, B^i = 0$)
- Initial Lapse
  - ▷ pre-collapsed $\alpha = \psi^{-2}$
  - ▷ Like UTB
  - ▷ No Instabilities if initially $\alpha = 1$, but the gauge pulse is smaller
  - ▷ Less dynamics as the gauge settles down in first few M

Unequal-Mass Simulations

Motivation:

- Supermassive Black Holes [LISA]
  - ▷ Kicks and Structure formation
- Stellar-Mass Black Holes: Structure in waveform? [LIGO]
  - ▷ Detection of GW signal more difficult
  - ▷ Parameter Estimation might be easier